NAME:

Spring 2020 Math 1201 Exam 3

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. Suppose that $f''(\theta) = \sin\theta + \cos\theta$, $f(0) = 3$, and $f'(0) = 4$. Find $f(\theta)$.
[10 pts]

2. Express the integral $\int_2^5 \left(3x + \sqrt{1+x^2}\right)dx$ as the limit of a Riemann sum of left rectangles L_n. Do not evaluate. [10 pts]

3. Set up as a definite integral

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(-4\left(2+k\frac{3}{n}\right)^3 + 2\left(2+k\frac{3}{n}\right) + 7\right)\frac{3}{n}.$$

Do not evaluate. [10 pts]

4. Compute the Riemann integral $\int_1^3 (2x-5)dx$ by expressing it as a limit of a Riemann sum and evaluating this limit. [No credit for integral 'shortcuts'] [10 pts]

5. Find the derivative of the function $F(x) = \int_0^{x^4} \cos^2\theta \, d\theta$ [10 pts]

6. Evaluate the limit $\lim_{n\to\infty} \frac{2}{n}\left(\left(1+\frac{2}{n}\right)^2 + \left(1+2\frac{2}{n}\right)^2 + \left(1+3\frac{2}{n}\right)^2 + \cdots + \left(1+n\frac{2}{n}\right)^2\right)$. You may apply the Fundamental Theorem of Calculus for ‘shortcuts’.

[10 pts]

7. Find the general indefinite integral of:

(a) $\int \frac{2x+\sqrt{x}}{x}dx$ [5 pts]

(b) $\int (1+\tan^2 \alpha)d\alpha$ [5 pts]

8. Evaluate $\int_{-5}^{0} \sqrt{25 - x^2} dx$ [10 pts]

9. Evaluate $\int_{0}^{1} (u + 2)(u - 3) du$ [10 pts]

10. Evaluate $\int_{-1}^{1} \sin(\pi x^3)\,dx$ Hint: Use symmetry [10 pts]

Extra-Credit

11. Prove the Fundamental Theorem of Calculus. Namely, prove that if $f : (\alpha, \beta) \to \mathbf{R}$ is continuous and $a \in (\alpha, \beta)$ is any point in the interval where $f(x)$ is defined, then $F(x) = \int_a^x f(t)dt$ is one of its antiderivatives. In particular, every continuous, real valued function has an antiderivative. [10 pts]

12. Let $f(x) = \begin{cases} \frac{\sin 5x}{x} & if\ x > 0 \\ -7 & if\ x = 0 \end{cases}$

What is $\lim_{h\to 0} \frac{1}{h}\int_0^h f(x)dx$? [10 pts]

13. If $x\sin(\pi x) = \int_0^{x^2} f(t)dt$, where *f* is a continuous function, find $f(4)$ [10 pts]